Surds, Indices & Proof

Q1.

Marking Instructions	AO	Marks	Typical Solution
Circles correct answer	1.1b	B1	$a^{\frac{3}{2}}$
			Total 1 mark

Q2.

(a) $\sqrt[4]{x^3} = x^{\frac{3}{4}}$

Accept k = ¾ OE

B1

1

(b) $\dfrac{1-x^2}{\sqrt[4]{x^3}} = \dfrac{1}{\sqrt[4]{x^3}} - \dfrac{x^2}{\sqrt[4]{x^3}} = x^{-k} - \dfrac{x^2}{\sqrt[4]{x^3}}$ $\left[\text{ or } \dfrac{1}{\sqrt[4]{x^3}} - x^{2-k}\right]$

Split followed by at least one correct index law used to remove denominator.

M1

$= x^{-\frac{3}{4}} - x^{\frac{5}{4}}$

If incorrect, ft on c's non-integer k value answer to part (a), provided M1 has been awarded. Accept answer given in form of values for p and q.

A1F

2

[3]

Q3.

Marking Instructions	AO	Marks	Typical Solution
Recalls that correct step is to multiply top and bottom by 2 – √n PI by subsequent work	1.2	M1	$\dfrac{3-\sqrt{n}}{2+\sqrt{n}} \times \dfrac{2-\sqrt{n}}{2-\sqrt{n}}$
Multiplies numerator and denominator by (2 - √n) to get correct terms (condone sign errors) Does not need to be simplified PI by correct simplification	1.1a	M1	$\dfrac{6-3\sqrt{n}-2\sqrt{n}+n}{4+2\sqrt{n}-2\sqrt{n}-n}$
Obtains correct simplified numerator and denominator	1.1b	A1	$\dfrac{6+n-5\sqrt{n}}{4-n}$

not necessarily in a fraction			
States correct expressions for a and b Or gives expression with a and b correctly identified	1.1b	A1	$a = \dfrac{6 + n}{4 - n}$ $b = \dfrac{-5}{4 - n}$
			Total 4 marks

Q4.

Marking Instructions	**AO**	**Marks**	**Typical Solution**
States algebraic expressions for two distinct non-consecutive odd numbers.	3.1a	M1	Let $n = 2p + 1$ $m = 2q + 1$ Where p and q are integers $m^2 + n^2 = (2p + 1)^2 + (2q + 1)^2$ $= 4p^2 + 4p + 1 + 4q^2 + 4q + 1$ $= 2(2p^2 + 2q^2 + 2p + 2q + 1)$ Factor 2 shows it is a multiple of 2 Factor $(2p^2 + 2q^2 + 2p + 2q + 1)$ is 1 more than a multiple of 2 so $m^2 + n^2$ is not a multiple of 4
Expands their two-termed expression for m and n in $m^2 + n^2$	1.1a	M1	
Obtains their correct expanded expression. Do not allow if substitutions define the same odd number.	1.1b	A1F	
Concludes correctly that the expression is a multiple of 2 Do not allow if substitutions define consecutive odd numbers or substitutions which generate the same odd number.	2.4	E1	
Completes a reasoned argument to conclude correctly the expression is not a multiple of 4. CAO OE Must not have used substitutions which involve m or n or define consecutive odd numbers or which generate the same odd number. CAO	2.1	R1	
			Total 5 marks

Coordinate Geometry

Q5.

	Marking Instructions	AO	Marks	Typical Solution
(a)	Completes the square twice or applies standard formula	AO1.1a	M1	$(x + 4)^2 + (y - 6)^2 - 16 - 36 = 12$ $(x + 4)^2 + (y - 6)^2 = 64$ Centre $(-4, 6)$ Radius = 8
	Obtains correct equation	AO1.1b	A1	
	Obtains correct radius and correct coordinates of *C* Follow through 'their' equation	AO1.1b	A1F	
(b)	Demonstrates a method to find the length *OP* or *OQ* (or their squares), or the coordinates *P* or *Q* using 'their' values from part **(a)**	AO3.1a	M1	$OC^2 = 4^2 + 6^2 = 52$ $OP^2 = r^2 - OC^2$ $= 64 - 52 = 12$ $PQ = 2OP$ $2\sqrt{12} = 4\sqrt{3}$
	Uses a circle property that may lead to a solution, eg radius and chord meet at right-angles (evidence for this could be the use of Pythagoras or perpendicular gradients)	AO3.1a	M1	
	Finds *OP* or *OQ* or coordinates of *P* or *Q* **CAO**	AO1.1b	A1	
	Obtains length of *PQ* Follow through from 'their' coordinates of *P* and *Q* (Does not need to be in the required form)	AO1.1b	A1F	
	Sets out a well-constructed mathematical argument, using precise statements and correct use of symbols	AO2.1	R1	

throughout to show the correct required result in required form Only award if they have a completely correct solution, which is clear, easy to follow and contains no slips			
			Total 8 marks

Q6.

	Marking Instructions	AO	Marks	Typical Solution
(a) (i)	Obtains centre = (3,4) Accept $a = 3$, $b = 4$	1.1b	B1	Centre (3,4)

(ii)	Rearranges into a standard form with $(x \pm 3)^2 + (y \pm 4)^2$ seen or Forms an expression for radius of the form $\sqrt{(\pm 3)^2 + (\pm 4)^2 \pm p}$ This working can be seen in part (a)(i)	1.1a	M1	$x^2 + y^2 = 6x + 8y + p$ $x^2 + y^2 - 6x - 8y - p = 0$ $(x - 3)^2 + (y - 4)^2 - 9 - 16 - p = 0$ $(x - 3)^2 + (y - 4)^2 = 25 + p$ Radius = $\sqrt{25 + p}$
	Obtains $(x - 3)^2 + (y - 4)^2 = 25 + p$ Or obtains $\sqrt{3^2 + 4^2 + p}$	1.1b	A1	
	States radius = $\sqrt{25 + p}$	1.1b	A1	

(b)	Begins to solve the problem by either Sketching (part of) a circle which goes through the origin or Sketching (part of) a circle that touches one of the axes or Substituting either $x = 0$ or $y = 0$ into their circle equation – must involve p PI by correctly formed equation involving p	3.1a	M1	Circle passes through the origin $\sqrt{25 + p} = 5$ $\Rightarrow p = 0$ Circle just touches x-axis $\sqrt{25 + p} = 4$ $\Rightarrow p = -9$
	Forms an equation to find p By either Equating their expression for	3.1a	M1	

the radius to 5 or the greater value of their a and b or Substituting both $x = 0$ and $y = 0$ into their circle equation to form an equation in terms of p only or Substituting $y = 0$ and using $b^2 - 4ac = 0$ to form an equation in terms of p only			
Deduces $p = 0$	2.2a	R1	
Deduces $p = -9$	2.2a	R1	
			Total 8 marks

Q7.

	Marking Instructions	AO	Marks	Typical Solution
(a)	Uses negative reciprocal to obtain an equation with the correct gradient.	1.1a	M1	$4y + 3x = c$ $4 \times 2 + 3 \times 15 = 53$ $4y + 3x = 53$
	Obtains correct equation ACF ISW once ACF seen Eg $y = -\frac{3}{4}x + \frac{53}{4}$ $y - 2 = -\frac{3}{4}(x - 15)$	1.1b	A1	

(b)	Begins to solve $3y - 4x = 21$ and their $4y + 3x = 53$ with elimination of one variable or better. Or obtains correct point of intersection for $3y - 4x = 21$ and their $4y + 3x = 53$	1.1a	M1	$3y - 4x = 21$ $4y + 3x = 53$ $y = 11$ $x = 3$ $(3 - 15)^2 + (11 - 2)^2 = 12^2 + 9^2$ $= 225$ Distance=15
	Uses distance formula to find the distance between (15, 2) and another point other than the origin. PI correct distance or square of correct distance	1.1a	M1	
	Uses distance formula for (15, 2) and their point of intersection to find distance or distance^2	3.1a	M1	

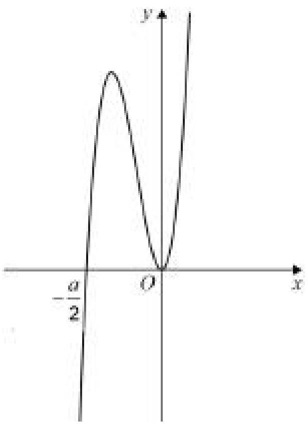
Obtains 15 CAO	1.1b	A1	
			Total 6 marks

Polynomials & Discriminant

Q8.

Marking Instructions	AO	Marks	Typical Solution
Recalls that the discriminant must be negative, seen anywhere in solution	1.2	B1	For no real solutions the discriminant must be negative
Substitutes 9, 4 and p^2 into the expression $b^2 - 4ac$ PI	1.1a	M1	$4^2 - 4 \times 9 \times p^2 < 0$
Deduces correct critical values of $\frac{2}{3}$ and $-\frac{2}{3}$	2.2a	A1	$p^2 > \frac{4}{9}$
Obtains two correct inequalities for p	2.5	A1	$p > \frac{2}{3}$ or $p < -\frac{2}{3}$
			Total 4 marks

Q9.

	Marking Instructions	AO	Marks	Typical Solution
(a)	Draws cubic curve in the correct orientation	1.1a	M1	
	Deduces minimum or maximum at (0,0) on their curve	2.2a	M1	
	Draws a fully correct cubic curve with x-intercept at $-\frac{a}{2}$ shown on the curve	2.2a	A1	

	Marking Instructions	AO	Marks	Typical Solution
(b)(i)	Substitutes $x = -3$ into p(x) Condone missing bracket for $(-3)^2$ Must see an expression in terms of a	1.1a	M1	$(-3)^2 (2 \times -3 + a) + 36 = 0$ $-54 + 9a + 36 = 0$ $9a - 18 = 0$ $a = 2$
	Completes reasoned argument with at least one correct intermediate step and no error seen to show $a = 2$ **AG** Must set an expression for p(−3) = 0 Condone recovery of missing	2.1	R1	

	bracket for $(-3)^2$ to get 9 Do not condone any other missing bracket			

(ii)	States 'translation' or 'translate' or 'translated' Must not have other transformation other than translation	1.1b	B1	Translation $\begin{pmatrix} 0 \\ 36 \end{pmatrix}$
	States the vector $\begin{pmatrix} 0 \\ 36 \end{pmatrix}$ or 36**j**	1.1b	B1	

(iii)	Explains that the translated graph only has one real solution or only has a root at -3 Condone missing 'real'	2.4	E1	The translated graph will only have one real solution. $b^2 - 4ac < 0$ Hence $b^2 - 4 \times 2 \times c < 0$ $b^2 < 8c$
	Deduces that the discriminant of $2x^2 + bx + c$ must be negative and shows the required result Do not allow the use of $a = 2$ with reference to part (b)(i) Allow $b^2 - 8c < 0$ following from $b^2 - 4ac$ seen	2.2a	E1	
				Total 9 marks

Trigonometry

Q10.

	Marking Instructions	AO	Marks	Typical Solution
(a) (i)	Uses identity to replace $\sin^2\theta$ with $(1 - \cos^2\theta)$ or uses $\sin\theta = \frac{\sqrt{3}}{2}$	AO1.2	M1	$6(1 - \cos^2\theta) + 5\cos\theta = 7$ $6\cos^2\theta - 5\cos\theta + 1 = 0$ $(2\cos\theta - 1)(3\cos\theta - 1) = 0$ $\cos\theta = \frac{1}{2}$
	Solves quadratic equation to get one solution $\cos\theta = \frac{1}{2}$ Or verifying using $\cos\theta = \frac{1}{2}$	AO1.1a	A1	
(ii)	States any two correct solutions	AO1.1b	B1	$\theta = 60°, 300°,$ Or $\cos\theta = \frac{1}{3}$ $\theta = 71°, 289°$
	States two additional correct solutions. Condone answers of 70.5 and 289.5 or greater accuracy. Ignore any additional answers outside the range but any additional answers inside range lose second B1	AO1.1b	B1	
(b)	Writes down a set that is half the values given as their solutions in part (a) Accept 36° or 144°	AO2.2a	M1	$\theta = 30°, 150°, 35°, 145°$ $210°, 330°, 215°, 325°$
	Writes down an additional set that is 180° more than the first set. Condone AWRT integer values	AO1.1b	A1F	
				Total 6 marks

Q11.

Marking Instructions	AO	Marks	Typical Solution
Uses substitution $\cos^2 x = 1 - \sin^2 x$ in any	1.2	B1	$9\sin^2 x - 6\sin x + (1 - \sin^2 x) = 0$

form			$8\sin^2 x - 6\sin x + 1 = 0$ $(4\sin x - 1)(2\sin x - 1)$ $\sin x = ¼$ $\sin x = ½$ 14°, 30°, 150°, 166°
Solves 'their' quadratic to obtain two values for $\sin x$	1.1a	M1	
Finds two correct solutions for x	1.1b	A1	
Finds all four solutions for x and no extras (condone 14.5, 165.5 AWRT)	1.1b	A1	
Total		**4**	

Q12.

$\tan \theta = -1$
$\sin^2 \theta = 3\cos^2 \theta$

B1

$\frac{\sin^2 \theta}{\cos^2 \theta} = \tan^2 \theta$

$\frac{\sin \theta}{\cos \theta} = \tan \theta$ used on $\sin^2 \theta - 3\cos^2 \theta$ or forms and solves a correct quadratic in sin or cos and then uses to find $\tan\theta$

M1

$\tan^2 \theta = 3$

$\tan^2 \theta = 3$ or $\tan^2 \theta - 3 = 0$
or $(\tan \theta + \sqrt{3})(\tan \theta - \sqrt{3}) = 0$
or $\tan \theta = \sqrt{3}$ or $\tan \theta = -\sqrt{3}$

A1

$\tan\theta = \pm \sqrt{3}$

Both

A1

[4]

Q13.

	Marking Instructions	AO	Marks	Typical Solution
(a)	Uses a trig identity, either $\tan\theta = \frac{\sin\theta}{\cos\theta}$ or $\sin^2 \theta + \cos^2 \theta = 1$	1.1a	M1	$5\cos^2 \theta = 4\sin^2 \theta$ $\frac{5}{4} = \frac{\sin^2\theta}{\cos^2\theta} = \tan^2\theta$

	correctly to obtain an equation in a single trig function.			$\tan\theta = \pm\dfrac{\sqrt{5}}{2}$
	Obtains $\tan^2\theta = \frac{5}{4}$ or $\sin^2\theta = \frac{5}{9}$ or $\cos^2\theta = \frac{4}{9}$ PI by one correct value for tan θ sin θ or cos θ	1.1b	A1	
	Obtains $\tan\theta = \pm\dfrac{\sqrt{5}}{2}$ OE Must be in exact form	1.1b	A1	
(b)	Obtains at least two correct solutions in the range based on the value of their tan θ, sin θ *or* cos θ OE	1.1a	M1	θ = 48.2, 131.8, 228.2, 311.8
	Obtains all 4 correct solutions and no further ones AWRT 48, 132, 228, 312	1.1b	A1	
				Total 5 marks

Differentiation & Integration

Q14.

Marking Instructions	AO	Marks	Typical Solution
Circles correct answer	AO1.1b	B1	$-\dfrac{1}{x\sqrt{x}}$
			Total 1 mark

Q15.

Marking Instructions	AO	Marks	Typical Solution
Substitutes $(x + h)$ into $f(x + h) - f(x)$ Condone one slip	1.1a	M1	
Obtains correct expanded expression for $f(x + h) - f(x)$	1.1b	A1	$\lim_{h\to 0}\left[\dfrac{3(x+h)-5(x+h)^2-(3x-5x^2)}{h}\right]$
Divides each term in their numerator by h	1.1a	M1	$\lim_{h\to 0}\left[\dfrac{3x+3h-5x^2-10xh-5h^2-3x+5x^2}{h}\right]$
Completes rigorous mathematical argument to show the required result. Must see $\lim_{h\to 0}$	2.1	R1	$\lim_{h\to 0}\left[\dfrac{3h-10xh-5h^2}{h}\right]$ $\lim_{h\to 0}[3-10x-5h]$ $\dfrac{dy}{dx}=3-10x$
			Total 4 marks

Q16.

(a) $\sqrt{x^5} = x^{\frac{5}{2}}$

Accept $k = 2.5$

B1

1

(b) $\displaystyle\int\left(7\sqrt{x^3}-4\right)dx = \frac{7}{3.5}x^{3.5} - 4x(+c)$

Index 'k' raised by 1 in integrating x^k

M1

1^{st} term correct follow through on non- integer k

A1F

For $-4x$ as integral of – 4

B1

3

(c) $y = 2x^{3.5} - 4x + c$ (*)

y = c's answer to (b) with '+ c'
('y =' PI by next line)

B1F

When $x = 1$, $y = 3 \Rightarrow 3 = 2 - 4 + c$

Subst. (1, 3) in attempt to find constant of integration

M1

$y = 2x^{3.5} - 4x + 5$

*Accept c = 5 after correct eqn * which must include 'y ='*
Coefficients must be tidied

A1

3

[7]

Q17.

(a) $\sqrt{x} = x^{\frac{1}{2}}$

PI

B1

$$\frac{x^3 + \sqrt{x}}{x} = \frac{x^3}{x} + \frac{\sqrt{x}}{x} = x^2 + x^{-\frac{1}{2}}$$

Accept p = 2; q = $-\frac{1}{2}$

B1;B1

3

(b) (i) $\frac{dy}{dx} = 2x - \frac{1}{2}x^{-\frac{3}{2}}$

Reduces both powers by 1

M1

ACF

A1

2

(ii) When $x = 1$, $y = 2$

PI if not stated explicitly eg the '2' may appear in the correct posn. in later eqn.

B1

When $x = 1$, $\frac{dy}{dx} = 2 - \frac{1}{2} = \frac{3}{2}$

Attempt to find $\frac{dy}{dx}$ *When* $x = 1$ *PI*

M1

Gradient of normal = $-\frac{2}{3}$

–1/ (c's value of dy/dx *when* $x = 1$*) either stated as the gradient of the normal or used as the gradient in the equation of the normal*

m1

Equation of normal: $y - 2 = -\frac{2}{3}(x - 1)$

Only ft on c's $\frac{dy}{dx}$ *in part (b)(i)*
ACftF

A1F

4

(c) (i) $\frac{d^2y}{dx^2} = 2 + \frac{3}{4}x^{-\frac{5}{2}}$

Reduces both powers by 1.

M1

Ft on (b)(i) provided at least one power to be differentiated is both negative and fractional

A1F

2

(ii) (Since $x > 0$,) $\frac{d^2y}{dx^2} > 0$

For a maximum point $\frac{d^2y}{dx^2}$ is **not** positive so C has no maximum points

E1 for attempt to find the sign of $\frac{d^2y}{dx^2}$ *;*
either in general terms or at the pt(s)

where c's $dy/dx = 0$ or the remaining E mark

a correct justification for why $\frac{d^2y}{dx^2} > 0$ *and also*
a full correct concluding statement must be made.

E2,1,0
2

Q18.

(a) (i) $3x^2 + 3x^2 + xy + xy + 3xy + 3xy$

correct expression for surface area

M1

$6x^2 + 8xy = 32 \Rightarrow 3x^2 + 4xy = 16$

$2(3x^2 + xy + 3xy) = 32$ etc

AG *be convinced*

A1
2

(ii) $(V =)\ 3x^2\, y$ OE

correct volume in terms of x and y

M1

$= 3x\left(\frac{16-3x^2}{4}\right)$ **or** $= 3x^2\left(\frac{16-3x^2}{4x}\right)$

OE

$= 12x - \frac{9x^3}{4}$

CSO AG

be convinced that all working is correct

A1
2

(b) $\left(\frac{dV}{dx} =\right) 12 - \frac{27}{4}x^2$

one of these terms correct

M1

all correct with 9×3 evaluated (no $+ c$ etc)

A1
2

(c) (i) $x = \frac{4}{3} \Rightarrow \frac{dV}{dx} = 12 - \frac{27}{4} \times \left(\frac{4}{3}\right)^2$

attempt to sub $x = \frac{4}{3}$ into 'their' $\frac{dV}{dx}$

M1

$\frac{dV}{dx} = 12 - \frac{27}{4} \times \frac{16}{9}$ = 12 − 12

or 12 − $\frac{432}{36}$ = 12 − 12 or 12 − $\frac{48}{4}$ = 0 etc

$\frac{dV}{dx}$ = 0 ⇒ stationary value

CSO; shown = 0 plus statement

A1

2

(ii) $\frac{d^2V}{dx^2} = -\frac{27x}{2}$ OE

FT for 'their' $\frac{dV}{dx}$ = $a + bx^2$

B1√

when $x = \frac{4}{3}$, $\frac{d^2V}{dx^2}$ < 0 ⇒ maximum

***or** sub of $x = \frac{4}{3}$ into 'their' $\frac{d^2V}{dx^2}$ ⇒ maximum*

(FT "*minimum*" if their $\frac{d^2V}{dx^2} > 0$)

EO if numerical error seen

E1√

2

[10]

Q19.

	Marking Instructions	AO	Marks	Typical Solution
(a)	Deduces one correct inequality related to the sloping line or the curve. Condone strict inequalities	2.2a	B1	$y \le x + 2$ $y \ge x^2 - 4x - 12$ $y \ge 0$
	Deduces the other two correct inequalities Condone strict inequalities	2.2a	B1	

(b)	States x coordinate of A is –2	1.1b	B1	A is (-2, 0) B is (6, 0) $x + 2 = x^2 - 4x - 12$ $x^2 - 5x - 14 = 0$ $(x + 2)(x - 7) = 0$
	States x coordinate of B is 6	1.1b	B1	
	Eliminates y to obtain x coordinate of $C = 7$	1.1a	M1	
	Obtains correct y coordinates	1.1b	A1	

	of *A*, *B* and *C*			*C* is point (7, 9)

(c)	Obtains correct value for area under *AC*	1.1b	B1	Area of triangle under *AC* $= 0.5 \times 9 \times 9$ $= 40.5$
	Integrates a quadratic $\frac{x^3}{3}$ term correct PI by $\frac{13}{3}$ ACF	1.1a	M1	Area below *BC*
	Integrates $x^2 - 4x - 12$ completely correct Condone inclusion of + *c* here PI by $\frac{13}{3}$ ACF Condone integration of $x^2 - 5x - 14$ correctly	1.1b	A1	$= \int_6^7 (x^2 - 4x - 12)\,dx$ $= [\frac{x^3}{3} - 2x^2 - 12x]_6^7$
	Substitutes a pair of limits into their integrated quadratic, must be three terms, including subtraction. PI by $\frac{13}{3}$ ACF	1.1a	M1	$= \frac{343}{3} - 98 - 84 - 72 + 72 + 72$
	Uses a correct method to combine areas that lead to the exact area of the shaded region	3.1a	M1	Shaded area $= 40.5 - 4\frac{1}{3}$
	Obtains $36\frac{1}{6}$ or $\frac{217}{6}$ ISW	2.1	R1	$= 36\frac{1}{6}$
				Total 12 marks

Q20.

	Marking Instructions	AO	Marks	Typical Solution
(a)	States correct coordinates of *B* or *C* Or States the correct x coordinates of B **and** C	1.1a	M1	*A* is (0, 2) *B* is (1, 0) *C* is (2, 0)
	Obtains A(0 , 2), B(1 , 0) and C(2 , 0)	1.1b	A1	

(b) (i)	Integrates in two parts with limits *O* to *B* and *B* to *C*	3.1a	M1	$\int_0^1 (x^2 - 3x + 2)\,dx$
	Integrates quadratic function with at least one term correct	1.1a	M1	$= [\frac{1}{3}x^3 - \frac{3}{2}x^2 + 2x]_0^1$
	Integrates completely correctly	1.1b	A1	
	Substitutes the two sets of their limits and subtracts the *BC*	1.1a	M1	$= [\frac{1}{3} - \frac{3}{2} + 2] = \frac{5}{6}$

value from the *OB* value or uses the modulus for the *BC* value			$\int_1^2 (x^2 - 3x + 2)\,dx$ $[\frac{1}{3}x^3 - \frac{3}{2}x^2 + 2x]_1^2$ $[\frac{1}{3} \times 8 - \frac{3}{2} \times 4 + 2 \times 2] - (\frac{5}{6})$ $= -\frac{1}{6}$ Total area = $\frac{5}{6} - (-\frac{1}{6}) = 1$
Completes calculation of total area convincingly to given answer. AG	2.1	R1	

(ii)	Explains that the area between B and C is treated as negative (OE)	2.3	E1	The calculator treats the area between B and C as negative.
				Total 8 marks

Exponentials & Logs

Q21.

(a) $b = a^c$

B1

1

(b) $2\log_2(x + 7) - \log_2(x + 5) = 3$

$\log_2(x + 7)^2 - \log_2(x + 5) = 3$

A law of logs used correctly on a correct expression.

M1

$\log_2 \dfrac{(x+7)^2}{x+5} = 3$

A further correct use of law of logs on a correct expression.

M1

$= 3\log_2 2 = \log_2 2^3$

$\Rightarrow \dfrac{(x+7)^2}{x+5} = 2^3$

$3 = 3\log_2 2$ or $3 = \log_2 2^3$ (= $\log_2 8$) seen
or
eg log f(x) = 3 ⇒ f(x) = 2^3 (= 8) OE

B1

$\Rightarrow (x + 7)^2 = 8(x + 5)$

Correct equation having eliminated logs and fractions

A1

$\Rightarrow x^2 + 14x + 49 = 8x + 40$

$\Rightarrow x^2 + 6x + 9\ (= 0)$

A1

Since $6^2 - 4(1)(9) = 0$, (there is only) one value of x (which satisfies the given equation).

OE

CSO Need conclusion which is also correctly justified

A1

6

[7]

Q22.

Marking Instructions	AO	Marks	Typical Solution
Takes logs of both sides and uses a log rule correctly	1.1a	M1	$ln\ 5^x = ln\ 3^{x+4}$ $x\ ln\ 5 = (x + 4)\ ln\ 3$ $x\ ln\ 5 - x\ ln\ 3 = 4\ ln\ 3$ $x(ln\ 5 - ln\ 3) = ln\ 81$ $x = \frac{ln\ 81}{ln\ 5 - ln\ 3}$
Applies all necessary log rules correctly so that x is no longer an exponent and expresses 4 ln 3 in terms of x Condone sign error	1.1a	M1	
Obtains ln 81 from 4 ln 3 or from $3^x \times 3^4$	1.1b	B1	
Completes reasoned argument to show given result Must see $x(ln\ 5 - ln\ 3)$ on the penultimate line If natural logs are not used throughout then the base must be converted at the end to get this mark	2.1	R1	
			Total 4 marks

Q23.

Marking Instructions	AO	Marks	Typical Solution
Takes logs to base 5 of both sides. Condone use of any base.	1.1a	M1	Take logs $2x + 4 = 2\log_5 9$ $2x + 4 = \log_5 3$ $x = -2 + \log_5 3$
Writes $\log_a 9$ as $2\log_a 3$ OE	1.1b	B1	
Obtains correct simplified answer PI by a = - 2 and b = 3	1.1b	A1	
			Total 3 marks

Q24.

(a) (i) $\log_a 40$

Accept 'k = 40'

B1

1

(ii) $\log_a 8$

Accept 'k = 8'

B1

1

(iii) $\log_a 125$

Accept 'k = 125' but not '$k = 5^3$'

B1

1

(b) $\log_{10} \{(1.5)^{3x}] = \log_{10} 7.5$

Correct statement having taken logs of both sides of $(1.5)^{3x} = 7.5$ OE PI or $3x = \log_{1.5} 7.5$ seen

M1

$3x \log_{10} 1.5 = \log_{10} 7.5$

$\log 1.5^{3x} = 3x \log 1.5$ OE

m1

$$x = \frac{\lg 7.5}{3 \lg 1.5} = 1.65645\ldots = 1.656 \text{ to } 3 \text{ dp}$$

Both method marks must have been awarded with clear use of logarithms seen

A1

3

(c) $\log_2 p = m \Rightarrow p = 2^m$; $\log_8 q = n \Rightarrow q = 8^n$

Either $p = 2^m$ or $q = 8^n$ seen or used

M1

$p = 2^m$ and $q = 2^{3n}$

Writing $8^n = 2^{3n}$ and having $p = 2^m$

m1

$pq = 2^m \times (2^3)^n = 2^m \times 2^{3n}$ so $pq = 2^{m+3n}$

Accept $y = m + 3n$

A1

3

[9]

[13]